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GEORGE C. MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA

SPIN REDUCTION FOR ION PROBE SATELLITE S-30 (19D)

by
Duane N. Counter

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ABSTRACT

This report presents an analysis of the methods of spin reduction of a satellite with a disc-like moment of inertia distribution. The two methods used reduce spin by the extension of masses on flexible wires. Phase 1 wires unwind freely around the periphery of the satellite and are released at the point of maximum extension. Phase 2 wires extend radially from the axis of spin and are restricted in their rate of travel by a braking device. Phase 2 wires are not disconnected from the Satellite after final extension.

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NOMENCLATURE

	UNITS	
d	in	first phase wire
I	in lb sec^2	mass moment of inertia of satellite without the spin reduction mass
m	lb $sec^2 in^{-1}$	total of n spin reduction masses. Refers to either first or second phase.
m_2	lb $sec^2 in^{-1}$	mass of the first phase wire
n		number of wires
r	in	second phase wire length
R	in	external radius of satellite
S_1	lb	force in the wire, first phase
S_2	lb	force in the wire, second phase
U_s	in sec^{-2}	acceleration of the first phase mass in the direction of the wire
V	in sec^{-1}	velocity of the first phase mass
V_u	in sec^{-1}	component of V perpendicular to the wire
X, Y	in	cartesian coordinates of one of the first phase masses
α		angular displacement of the first phase wire from the radial
γ		angle between phase 1 wire and a tangent to surface of satellite (Fig. 6).
θ		angle between phase 1 wire and a radial line through the axis of spin and the mass (Fig. 5).
ρ	in	displacement vector of the first phase mass from the spin axis

ω	sec^{-1}	angular velocity of the satellite
ω_o	sec^{-1}	velocity before first phase spin reduction
ω_1	sec^{-1}	velocity during first phase spin reduction when $\alpha = 90^o$
ω_2	sec^{-1}	velocity at the end of the first phase spin reduction and start of the second phase
ω_3	sec^{-1}	velocity at the end of the second phase spin reduction
Ω		angle between a radial line to the attaching point of the wire and the Y space fixed axis (Fig. 6)

SECTION I. PRELIMINARY DESIGN CONSIDERATIONS

Payload S-30 is a satellite with a double cone outside configuration and a disc-like moment of inertia distribution. The spin moment of inertia is about 30 in lb sec^2. The satellite is to be launched by a JUNO II vehicle with the satellite and upper stages spinning at 450 rpm. After injection into orbit, the spin must be reduced to 25 rpm to meet the requirements of the experiments.

For maximum spin lifetime it is desirable to reduce the spin through conservation of angular momentum. This can be accomplished by affixing masses to the extremities of the extendable dipole antenna and thus reducing the spin by increasing the moment of inertia. As spin reduction from 450 rpm to 25 rpm would require masses of 5.7 lbs weight at an antenna length of ten feet, weight limitations call for the use of another method in conjunction with this, thus reducing the spin in two phases.

The method chosen for the first phase consists of two wires wound around the circumference of the satellite in a direction opposite to the direction of the spin. Masses are affixed to the free ends and the wires are permitted to unwind. When the wires are fully unwound they are released to prevent their winding back up and restoring the spin to its initial value. This method represents an optimization of spin reduction with regard to weight saving but has the disadvantage of reducing the angular momentum of the satellite and thereby reducing its spin life.

SECTION II. SPIN REDUCTION FOR S-30

1. Basic Equations

The angular velocity ratio of the satellite at the end of the first phase of spin reduction is expressed by the equation:

$$\frac{\omega_2}{\omega_o} = \frac{\frac{I}{m} + R^2}{\frac{I}{m} + (R+d)^2} \left[1 - (R+d) \sqrt{\frac{2\ Rd + d^2}{\frac{I}{m}\left(\frac{I}{m} + R^2\right)}} \right] \tag{1}$$

This equation can be transformed giving the wire length as a function of the spin reduction and the moment of inertia to mass ratio. A plot of that transformation is given in Figure 1.

The wire force for the first phase is:

$$S_1 = \frac{I}{n} \left[\frac{R\omega^2 \cos \alpha + d (\omega - \dot{\alpha})^2}{\frac{I}{m} + R^2 \sin^2 \alpha} \right] \quad (2)$$

where $$\frac{\dot{\alpha}}{\omega_o} = \frac{\frac{\omega}{\omega_o} (d + 2 R \cos \alpha)}{d + R \cos \alpha} + \frac{\left(\frac{\omega}{\omega_o} - 1\right)\left(\frac{I}{m} + R^2\right)}{d (d + R \cos \alpha)}$$

The angular velocity ratio of the satellite at the maximum extension of the weights during the second phase is:

$$\frac{\omega_3}{\omega_2} = \frac{\frac{I}{m} + R^2}{\frac{I}{m} + (R + r)^2} \quad (3)$$

which is plotted in Figure 2 for the general case. The wire force for the second phase is:

$$S_2 = \frac{m}{n} \omega^2 (r + R) \quad (4)$$

The maximum value of S_2 is:

$$S_2 \max = \frac{9}{16} \omega^2 \frac{m}{n} \left(1 + \frac{R^2}{\frac{I}{m}}\right)^2 \sqrt{\frac{\frac{I}{m}}{3}} \quad (5)$$

This curve is plotted for the general case in Figure 3.

2. Design

From an examination of Figure 1, it is seen that as spin reduction increases due to a change in any one of the physical parameters, its accuracy decreases. Also, since the first phase throws away angular momentum, it reduces the total spin life of the satellite. It is, therefore, desirable to minimize the portion of the spin reduction performed by this phase. From the standpoint of weight, however, it is desirable to maximize the first phase portion.

Using a cross over between the two phases of 100 rpm, Figure 1 indicates a wire length of 200 inches would be favorable. Physical location of sensors along the rim of the satellite, dictates a wire length of 206.03 inches. Using this value and 30 in lb sec^2 for the moment of inertia, the total mass required is 0.150 lb. Using the antenna length of 120 inches for the second phase spin reduction, a second phase mass of 2.00 lb. results.

SECTION III. ANALYSIS OF FIRST PHASE DESIGN

For the case where $\alpha = 90°$, it is $\dot{\alpha} = -\omega_0$ and $d = \omega_0 Rt$. Then Equations 1 and 2 give wire force and velocity as a function of time. When $\alpha < 90°$ Equations 8 and 2 give wire force and velocity as a function of α. If it is assumed that $\alpha \approx -\omega_0 t$ + constant, this data can be converted to time scale. Plots of these equations are given in Figure 4.

1. Effect of Distributed Mass

An analysis of the effect of the distributed mass of the wire on the spin reduction may be made by considering the initial and final release conditions only. By making an analysis similar to the basic analysis and integrating the effects along the wire, a very similar equation results. If the mass at the end of the wire is much larger than the mass of the wire and if the wire length is larger than the radius of the payload, certain small terms may be neglected giving Equation 1 with m equal to the sum of the concentrated mass plus one third the mass of the wire.

The effect of the distributed mass on the wire force may be found approximately by assuming all rotation is around the center of spin. The effect of the wire may be found by integrating, using a corrected value of angular velocity in the assumption so that the force on the concentrated mass is the correct one from Equation 2. This results in a force of

$$F = S_1 \left(1 - \frac{m_2}{6m}\right)$$

where m_2 is the mass of the wire and S_1 is the value for wire force found from Equation 2.

2. Effect of Unsynchronized Initial or Final Release

All previous derivations have assumed symmetrical spin with two or more wires. If the wires are unwound unsymmetrically, an equation similar to the basic equation may be derived for each wire using the wire force derivation to find the angular acceleration of the body. It was found that, while the angle of deviation increases as the wires unwind, its rate of increase is small if the initial angle is not too large. A good approximation for determining the angle increase is to assume each wire has a constant angular velocity relative to the payload and equal to the angular velocity of the payload at the initial release of the masses. When the second mass is released, the first mass has already reduced the spin so the second mass unwinds with smaller angular velocity than the first. The difference in velocities may be used to find the increase in the angle of deviation. Some representative deviation angles and their increases are

INITIAL	INCREASE
30°	0.6°
90°	5.1°
180°	20.0°

The time required for a 30° angle of deviation is 0.011 seconds so the normal accuracy of the squibs used for the initial release should be quite adequate.

The effect of unsynchronized final release is negligible. More important is the effect of the final release being at some angle other than perpendicular to the surface of the satellite. This effect can be seen from the plot of ω in Figure 4. **Since the last .05 sec contain all 90°, a small angle deviation will affect only the very last of the velocity curve. The effect of the deviation goes approximately as 1 minus cosine of the deviation angle and is symmetrical. Figure 4 or Equation 8 may be used to find the deviation permissible for a given velocity deviation.**

3. Effect of Stretch of the Wire and Temperature Expansion

The force in the wire may be used to calculate stretch and that in turn in Equation 1 to find a modified spin reduction. For small elongations the change in final first phase spin is approximately $\Delta \omega_2 = 1.8\ \delta$ rpm where δ is the deflection due to the force on the wire. For a wire diameter of 0.031 in and steel, the velocity change is

$$\Delta \omega_2 = 0.29 \text{ rpm}$$

The effect on spin due to a change in temperature causing an expansion or contraction of the wire is, for a 120°F change,

$$\Delta \omega_2 = 1.14 \text{ rpm}$$

4. Accuracy Requirements for Moment of Inertia, Mass and Wire Length

Equation 1 may be used to find the change in ω_2 with a given change in I/m or d.

The following variations in ω_2 result from the accompanying changes in I/m.

I/m	ω_2
.7%	.36 rpm
2.0%	3 rpm
10.0%	20 rpm

Wire length variations are the same as the effect of the stretch of the wire; namely

$$\Delta\omega_2 = 1.88 \text{ rpm}$$

The effect of the wire force acting on the satellite under a nutation angle of 20° was investigated and the increase in the angle of nutation was found to be less than 3 minutes.

SECTION IV. ANALYSIS OF SECOND PHASE DESIGN

Using Equations 3 and 4, spin reduction and wire force may be calculated as a function of the wire length. These are plotted in Figure 5.

1. Effect of Distributed Mass

By writing a conservation of momentum equation including the distributed mass of the wire, the result is Equation 3 with m equal to the mass at the end plus one third the mass of the wire.

The effect of the distributed mass on the wire force may be determined by writing Equation 4 for an element of the wire and integrating to give Equation 4 where m is the mass at the end plus one half the mass of the wire.

2. Effect of Stretch of the Wire

Using the force to calculate the stretch in the wire and the stretch to calculate the change in spin reduction; the result is a negligible change in spin reduction because of the small final spin and final force.

3. Forces Caused by Final Release of the Weights

The weights are allowed to fall free for the last 1.375 inches and the impact due to the final stopping of the movement of the mass relative to the payload causes considerable stress in the second phase wire.

By solving for the velocity of the mass relative to the payload and in a direction along the wire, the kinetic energy in the mass that has to be dissipated by impact may be calculated. In this case the energy is 3.49 in lb for each weight. **If it is assumed that stress is linear with strain and if a wire with a glass core of an effective diameter of 0.13 in is used, the maximum force is 88 lb.**

4. Accuracy Requirements for Moment of Inertia, Mass and Wire Length

Equation 3 may be solved for I, m or r as a function of spin and used to find the variation for a given change in spin. The change in final velocity due to a one percent variation in I/m is 0.18 rpm and due to a one percent change in wire length is 0.33 rpm.

Summing up all the effects a tabulation may be made as follows:

FIRST PHASE

Element	Variation	$\Delta\omega_2$	$\Delta\omega_3$
I	± 1 in. lb. sec^2		
m	± 0.001 lb		
I/m	± 1%	± 1 rpm	± .25 rpm
d	± 0.25 in	± .45 rpm	± .11 rpm
Temperature	± 120°F	± 1.14 rpm	± .29 rpm
TOTAL		± 2.59 rpm	± .65 rpm

SECOND PHASE

m	± 0.013 lb	
I/m	± 1%	± 0.18 rpm
r	± 0.25 in	± 0.07 rpm
TOTAL		± 0.25 rpm
	First plus second	± 0.90 rpm

APPENDIX

The equations for the first phase spin reduction may be derived by assuming no friction and uniform unreeling of the wires. In this case, equations for conservation of angular momentum and conservation of energy may be written. Referring to Figure 6 since angular momentum is conserved,

$$\frac{d}{dt}\left[I\omega + m\rho\ (\rho\omega + V_u \cos\theta)\right] = 0 \tag{6}$$

where $\rho\omega + V_u \cos\theta$ is the component of the velocity V of the mass in a direction perpendicular to the distance vector ρ.

By conservation of energy

$$\frac{d}{dt}\left[\frac{1}{2} I\omega^2 + \frac{1}{2} mV^2\right] = 0 \tag{7}$$

By trigonometry from Figure 6, Equations 6 and 7 may be transformed and solved for ω to give

$$\frac{\omega}{\omega_0} = \frac{\frac{I}{m} + R^2}{\frac{I}{m} + R^2 + 2Rd\cos\alpha + d^2} \cdot \left[1 - \sqrt{1 - \frac{\left(\frac{I}{m} + d^2 + 2Rd\cos\alpha + R^2\right)\left(\frac{I}{m} + R^2\sin^2\alpha - d^2 - 2Rd\cos\alpha\right)}{\left(\frac{I}{m} + R^2\sin^2\alpha\right)\left(\frac{I}{m} + R^2\right)}}\right] \tag{8}$$

For the special case where $\alpha = 90°$, $\omega = \omega_1$

$$\frac{\omega_1}{\omega_0} = \frac{\frac{I}{m} + R^2 - d^2}{\frac{I}{m} + R^2 + d^2} \tag{9}$$

This is the equation used for the time when the wire is unwinding tangent to the body and the wire length d is changing.

When $\alpha = 0°$, $\omega = \omega_2$

$$\frac{\omega_2}{\omega_0} = \frac{\frac{I}{m} + R^2}{\frac{I}{m} + (R+d)^2}\left[1 - (R+d)\sqrt{\frac{2Rd + d^2}{\frac{I}{m}\left(\frac{I}{m} + R^2\right)}}\right] \tag{10}$$

Also, the time rate of change of α may be found from $\dot{\alpha} = -\frac{V_u}{d}$ to be

$$\frac{\dot{\alpha}}{\omega_o} = \frac{\omega/\omega_o \left(d + 2R \cos \alpha\right)}{d + R \cos \alpha} + \frac{\left(\omega/\omega_o - 1\right)\left(\frac{I}{m} + R^2\right)}{d\,(d + R \cos \alpha)} \qquad (11)$$

If a space fixed coordinate system is drawn as in Figure 7, expressions may be written for the displacement of the masses.

$$X = -R \sin \Omega + d \cos (\gamma + \Omega)$$

$$Y = R \cos \Omega + d \sin (\gamma + \Omega)$$

These expressions may be differentiated twice to find the X and Y components of acceleration. With trigonometry, the components of acceleration perpendicular to and parallel to the wire d may be found. Obviously, the perpendicular component is zero and the parallel component will give the force in the wire.

The parallel component is:

$$\ddot{U}_s = -R\omega^2 \cos \alpha - R\dot{\omega} \sin \alpha - d(\omega - \dot{\alpha})^2$$

The wire force is then: $S_1 = -\frac{m}{n}\ddot{U}_s$ where n is the number of wires, then

$$S_1 = \frac{I}{n}\left[\frac{R\omega^2 \cos \alpha + d(\omega - \dot{\alpha})^2}{\frac{I}{m} + R^2 \sin^2 \alpha}\right] \qquad (12)$$

The second phase spin reduction equations may be derived by conservation of angular momentum, (see Figure 8)

$$\frac{d}{dt}\left[I\omega + m(R + r)^2 \omega\right] = 0$$

which gives:

$$\frac{\omega}{\omega_2} = \frac{\frac{I}{m} + R^2}{\frac{I}{m} + (R + r)^2} \qquad (13)$$

The wire force in the second phase is

$$S_2 = \frac{m}{n}\omega^2 (R + r) \qquad (14)$$

If 14 is differentiated with respect to r and equated to zero, the maximum value of wire force will result.

$$S_2 \max = \frac{9}{16} \omega_2{}^2 \frac{m}{n} \left(1 + \frac{R^2}{\frac{I}{m}}\right)^2 \sqrt{\frac{\frac{I}{m}}{3}} \tag{15}$$

where $\sqrt{\frac{\frac{I}{m}}{3}}$ is the distance at which this occurs.

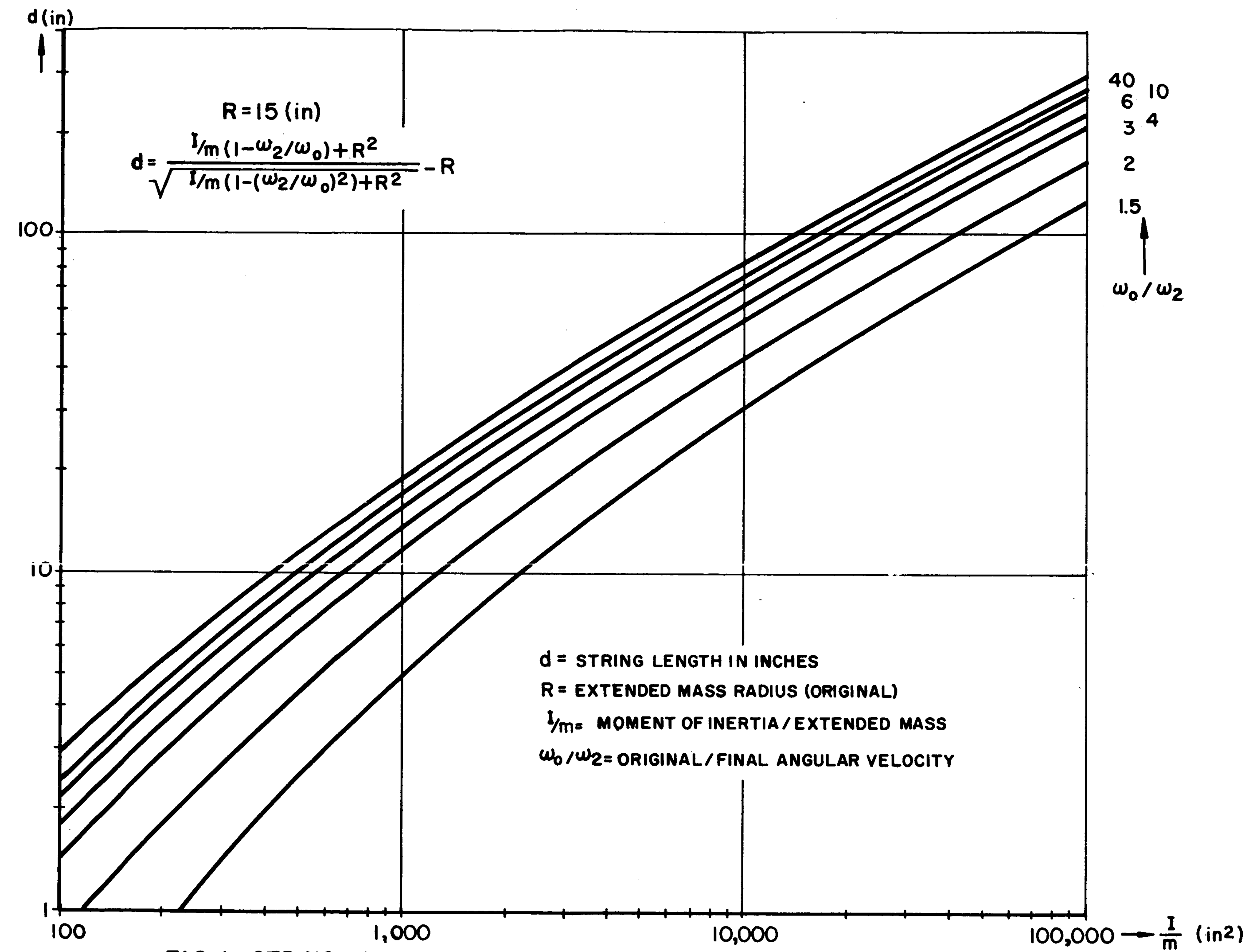

FIG. 1 STRING LENGTH VS RATIO OF MOMENT OF INERTIA TO EXTENDED MASS FOR VARIOUS SPIN REDUCTION RATIOS
FIRST PHASE SPIN REDUCTION

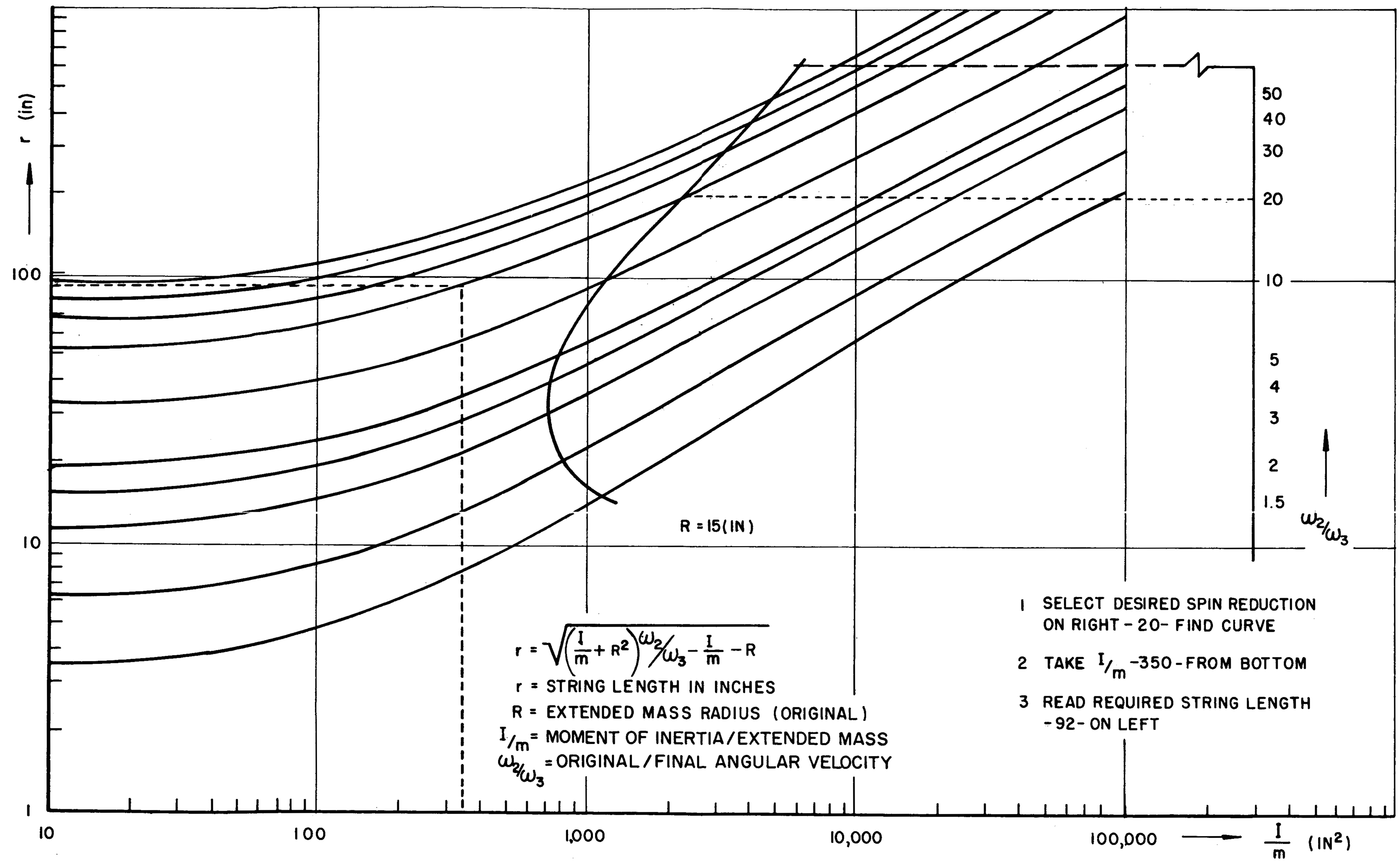

FIG. 2 STRING LENGTH VS. MOMENT OF INERTIA TO EXTENDED MASS RATIO FOR VARIOUS SPIN REDUCTION RATIOS
SECOND PHASE SPIN REDUCTION

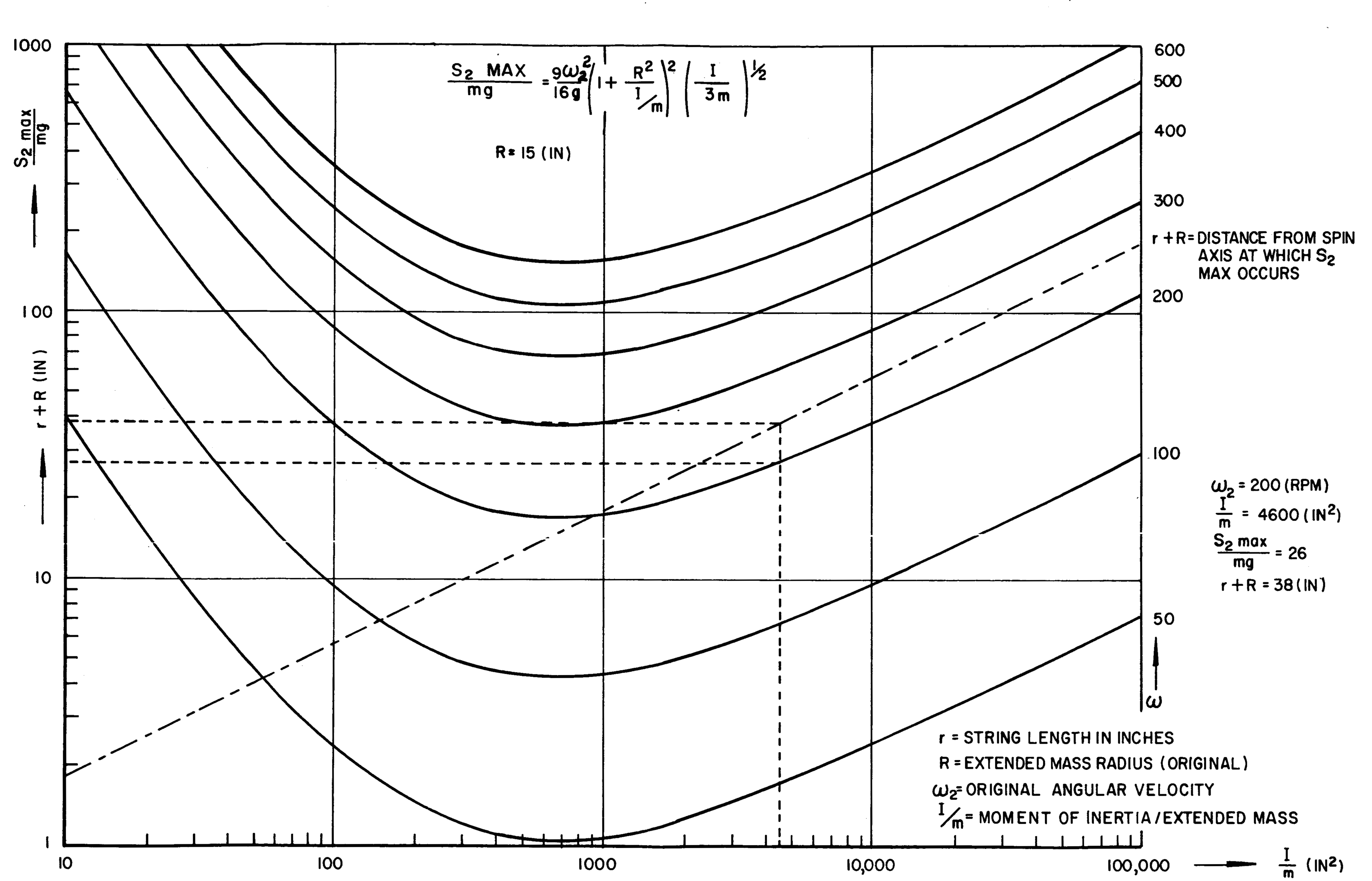

FIG. 3 MAXIMUM WIRE FORCE, SECOND PHASE SPIN REDUCTION

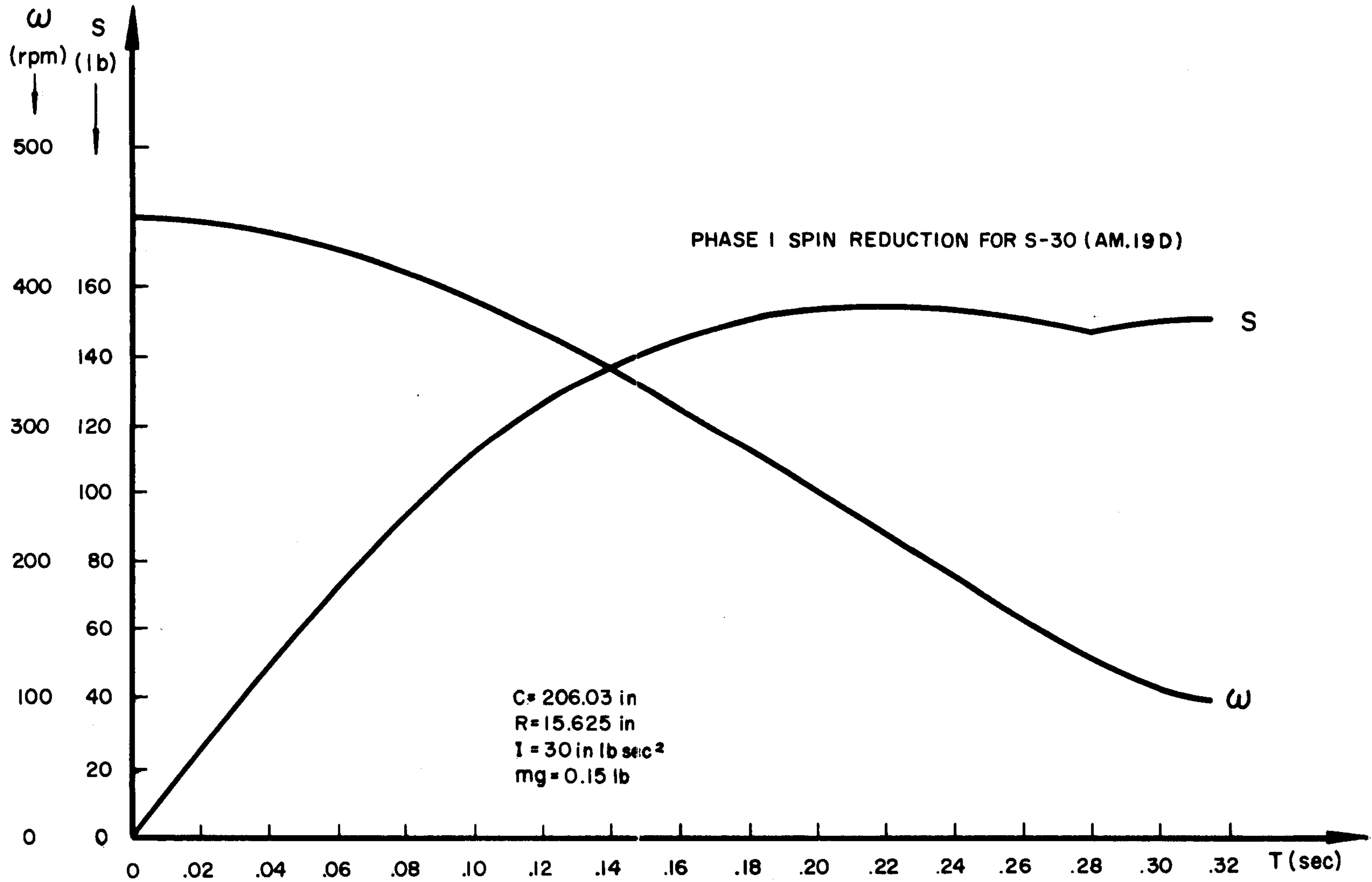

FIG. 4 WIRE FORCE AND ANGULAR VELOCITY VS TIME AFTER RELEASE

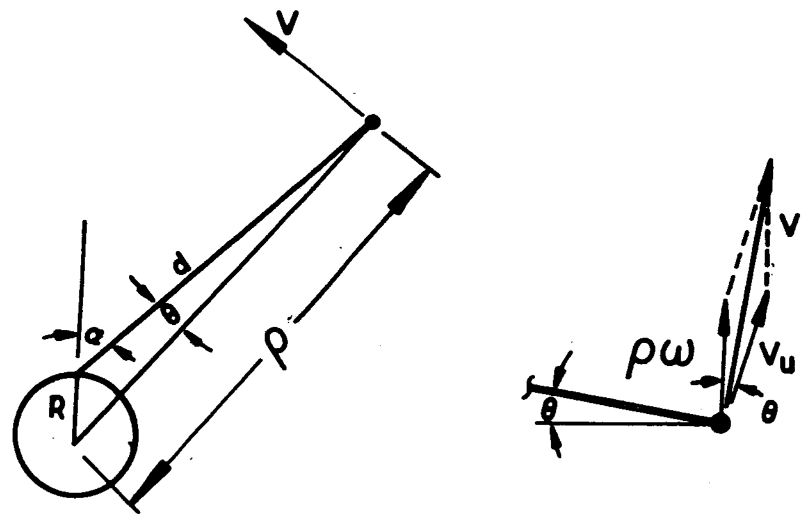

FIGURE 6

VELOCITY DIAGRAM
PHASE I

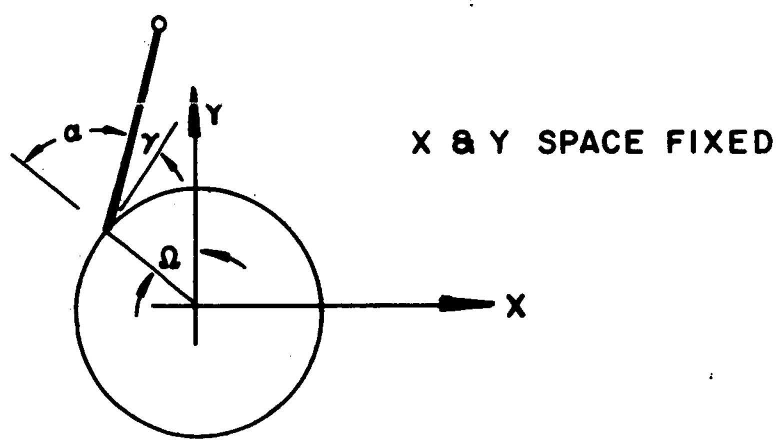

FIGURE 7

SPACE FIXED DIAGRAM
PHASE I

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